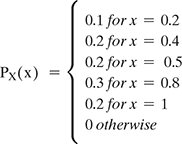
1. **Given X be a discrete random variable with the following PMF**

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**1. Find the range RX of the random variable X.**

**2. Find P(X ≤ 0.5)**

**3. Find P(0.25<X<0.75)**

**4. P(X = 0.2|X<0.6)**

**A.**

1. \*\*Finding the range \( R\_X \) of the random variable \( X \)\*\*:

The range of a random variable is the set of all possible values it can take. Since you have a discrete random variable with a probability mass function (PMF), we need to identify all possible values of \( X \) for which the PMF is defined. Once we have these values, we have the range \( R\_X \).

2. \*\*Finding \( P(X \leq 0.5) \)\*\*:

To find this probability, we sum up the probabilities of all values of \( X \) that are less than or equal to 0.5 according to the PMF.

3. \*\*Finding \( P(0.25 < X < 0.75) \)\*\*:

To find this probability, we sum up the probabilities of all values of \( X \) that lie between 0.25 and 0.75, inclusive.

4. \*\*Finding \( P(X = 0.2 | X < 0.6) \)\*\*:

This is a conditional probability. We want to find the probability of \( X \) being 0.2 given that \( X \) is less than 0.6. We use the conditional probability formula to calculate this.

Let's start by finding the range of \( X \). If you provide the PMF, I can assist you further with the calculations.

2. **Two equal and fair dice are rolled, and we observed two numbers X and Y.**

**1. Find RX, RY, and the PMFs of X and Y.**

**2. Find P(X = 2,Y = 6).**

1. **Find P(X>3|Y = 2).**

A. 1. \*\*Find RX, RY, and the PMFs of X and Y:\*\*

The random variables RX and RY represent the outcomes of the two dice rolls. Since each die has 6 sides with numbers from 1 to 6, RX and RY can take values from 2 to 12.

To find the PMFs (Probability Mass Functions) of X and Y, we need to determine the probabilities of each outcome. Since both dice are fair, each outcome has an equal probability of 1/36.

For RX and RY:

- RX takes values from 2 to 12, each with a probability of 1/36.

- RY takes values from 2 to 12, each with a probability of 1/36.

2. \*\*Find P(X = 2,Y = 6):\*\*

The probability of both dice showing X = 2 and Y = 6 simultaneously is the same as the probability of getting a 2 on the first die and a 6 on the second die. Since the rolls of each die are independent events, we can multiply the probabilities of each event occurring:

\[ P(X = 2, Y = 6) = P(X = 2) \times P(Y = 6) \]

\[ = \frac{1}{36} \times \frac{1}{36} \]

\[ = \frac{1}{1296} \]

3. \*\*Find P(X>3|Y = 2):\*\*

This is asking for the conditional probability that X is greater than 3 given that Y is 2. Since we know Y = 2, we only need to consider the outcomes where Y = 2. Then, we find the probability that X is greater than 3 among these outcomes.

The outcomes where Y = 2 are (2, 2), (3, 2), (4, 2), (5, 2), and (6, 2). Out of these, the outcomes where X > 3 are (4, 2), (5, 2), and (6, 2).

So, the conditional probability is:

\[ P(X > 3 | Y = 2) = \frac{\text{Number of outcomes where } X > 3 \text{ and } Y = 2}{\text{Total number of outcomes where } Y = 2} \]

\[ = \frac{3}{5} \]

1. **If Z = X + Y. Find the range and PMF of Z.**

**A.** To find the range and probability mass function (PMF) of \( Z = X + Y \), where \( X \) and \( Y \) are two random variables, we need to know the distributions of \( X \) and \( Y \). Let's assume \( X \) and \( Y \) are discrete random variables with PMFs denoted by \( p\_X(x) \) and \( p\_Y(y) \), respectively.

The range of \( Z \) will be all possible values that \( Z \) can take.

For the PMF of \( Z \), denoted as \( p\_Z(z) \), it's the probability that \( Z \) takes on a specific value \( z \). It's calculated by convolving the PMFs of \( X \) and \( Y \):

\[ p\_Z(z) = \sum\_{x} p\_X(x) \cdot p\_Y(z - x) \]

Where \( p\_Y(z - x) \) denotes the probability that \( Y \) takes on the value \( z - x \), given that \( X = x \).

Let's say \( X \) can take values \( x\_1, x\_2, \ldots, x\_n \) with probabilities \( p\_X(x\_1), p\_X(x\_2), \ldots, p\_X(x\_n) \) respectively, and similarly for \( Y \). Then the range of \( Z \) will be the set of all possible sums \( x\_i + y\_j \), where \( x\_i \) and \( y\_j \) are values taken by \( X \) and \( Y \) respectively.

And the PMF of \( Z \) will be computed as described above for each possible \( z \) in the range of \( Z \).

Could you provide the distributions of \( X \) and \( Y \) (either as a table or as a formula) so I can help you compute the range and PMF of \( Z \)?

1. **Find P(X = 4|Z = 8).**

A. To find \( P(X = 4 | Z = 8) \), where \( X \) and \( Z \) are random variables, you can use the conditional probability formula:

\[ P(X = 4 | Z = 8) = \frac{P(X = 4 \cap Z = 8)}{P(Z = 8)} \]

If \( X \) and \( Z \) are independent, then \( P(X = 4 \cap Z = 8) = P(X = 4) \cdot P(Z = 8) \).

However, if they are dependent, you would need additional information about their relationship to calculate \( P(X = 4 \cap Z = 8) \).

Without information about the relationship between \( X \) and \( Z \), I can't determine \( P(X = 4 | Z = 8) \) directly.

3. **In an exam, there were 20 multiple-choice questions. Each question had 44 possible options. A student knew the answer to 10 questions, but the other 10 questions were unknown to him, and he chose answers randomly. If the student X's score is equal to the total number of correct answers, then find out the PMF of X. What is P(X>15)?**

**A.** To find the probability mass function (PMF) of the random variable \( X \), which represents the score of the student, we need to determine the probabilities of each possible score from 0 to 20.

Given that the student knew the answer to 10 questions and randomly guessed the rest, we can calculate the PMF as follows:

Let's denote:

- \( n = 20 \) (total number of questions)

- \( k = 10 \) (number of questions the student knew the answer to)

- \( p = \frac{1}{44} \) (probability of guessing the correct answer to one question)

We can use the binomial probability formula to calculate the PMF:

\[ P(X = x) = \binom{n}{x} \times (p^x) \times ((1-p)^{n-x}) \]

Where:

- \( \binom{n}{x} \) is the number of combinations of choosing \( x \) correct answers out of \( n \) questions.

- \( p^x \) is the probability of getting \( x \) correct answers.

- \( (1-p)^{n-x} \) is the probability of getting \( n-x \) incorrect answers.

For \( x \) from 0 to 20, the PMF is:

\[ P(X = 0) = \binom{20}{0} \times \left(\frac{10}{44}\right)^0 \times \left(1-\frac{10}{44}\right)^{20} \]

\[ P(X = 1) = \binom{20}{1} \times \left(\frac{10}{44}\right)^1 \times \left(1-\frac{10}{44}\right)^{19} \]

\[ P(X = 2) = \binom{20}{2} \times \left(\frac{10}{44}\right)^2 \times \left(1-\frac{10}{44}\right)^{18} \]

\[ \vdots \]

\[ P(X = 20) = \binom{20}{20} \times \left(\frac{10}{44}\right)^{20} \times \left(1-\frac{10}{44}\right)^0 \]

After calculating these probabilities, you'll get the PMF for \( X \).

To find \( P(X > 15) \), you would sum the probabilities for \( X = 16, 17, 18, 19, 20 \).

4**. The number of students arriving at a college between a time interval is a Poisson random variable. On average, 10 students arrive per hour. Let Y be the number of students arriving from 10 am to 11:30 am. What is P(10<Y≤15)?**

A. To solve this problem, we can use the properties of the Poisson distribution. Given that the average number of students arriving per hour is 10, we need to find the probability of having between 11 and 15 students arrive between 10 am and 11:30 am.

First, let's calculate the average number of students arriving between 10 am and 11:30 am:

The time interval from 10 am to 11:30 am is 1.5 hours.

So, the average number of students arriving between 10 am and 11:30 am is:

Average number of students = (Average number of students per hour) × (Time in hours)

= 10 students/hour × 1.5 hours

= 15 students

Now, we can use the Poisson distribution formula to calculate the probability:

P(Y = y) = (e^(-λ) \* λ^y) / y!

Where:

- λ is the average number of arrivals in the given time period,

- y is the number of arrivals we're interested in, and

- e is Euler's number, approximately equal to 2.71828.

We want to find P(10 < Y ≤ 15). This is the cumulative probability of having more than 10 arrivals but less than or equal to 15 arrivals.

So, we need to sum the probabilities for y = 11, 12, 13, 14, and 15.

P(10 < Y ≤ 15) = P(Y = 11) + P(Y = 12) + P(Y = 13) + P(Y = 14) + P(Y = 15)

Using the Poisson distribution formula with λ = 15, we can calculate each probability and then sum them up.

Let's calculate each term:

For y = 11:

P(Y = 11) = (e^(-15) \* 15^11) / 11!

For y = 12:

P(Y = 12) = (e^(-15) \* 15^12) / 12!

For y = 13:

P(Y = 13) = (e^(-15) \* 15^13) / 13!

For y = 14:

P(Y = 14) = (e^(-15) \* 15^14) / 14!

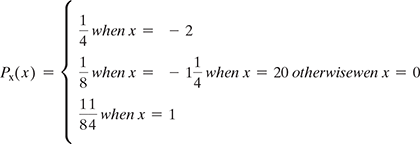
For y = 15:

P(Y = 15) = (e^(-15) \* 15^15) / 15!

Once you have calculated these probabilities, sum them up to get P(10 < Y ≤ 15).

5.Two independent random variables, X and Y,are given such that X~Poisson(α) and Y~Poisson(β). State a new random variable as Z = X + Y. Find out the PMF of Z.

6. **There is a discrete random variable X with the pmf.**

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**If we define a new random variable Y = (X + 1)2 then**

**1. Find the range of Y.**

**2. Find the pmf of Y.**

A. Sure, let's break it down step by step:

1. \*\*Finding the range of Y\*\*:

If \( Y = (X + 1)^2 \), then \( Y \geq 0 \) since it's a square. Now, let's find the maximum value of Y:

Since \( X \) is a discrete random variable, it can take on integer values. Let's say the maximum value of \( X \) is \( M \). Then, the maximum value of \( Y \) occurs when \( X = M \):

\( Y = (M + 1)^2 = M^2 + 2M + 1 \)

So, the range of \( Y \) is all non-negative integers from \( 0 \) to \( M^2 + 2M + 1 \).

2. \*\*Finding the pmf of Y\*\*:

To find the pmf of \( Y \), we need to find the probability that \( Y \) takes on each value in its range. Since \( Y = (X + 1)^2 \), we'll use the relationship between the pmfs of \( X \) and \( Y \).

Let \( p\_X(x) \) be the pmf of \( X \). Then, the pmf of \( Y \) can be calculated as follows:

\( P\_Y(y) = P((X+1)^2 = y) = P(X+1 = \sqrt{y}) + P(X+1 = -\sqrt{y}) \)

We know that \( X \) takes on integer values, so \( \sqrt{y} \) must also be an integer. Thus, \( y \) must be a perfect square. Let \( y = k^2 \) where \( k \) is a non-negative integer.

Therefore,

\( P\_Y(y) = P(X+1 = k) + P(X+1 = -k) \)

Finally,

\( P\_Y(y) = p\_X(k-1) + p\_X(-k-1) \)

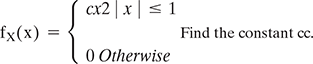
This gives us the pmf of \( Y \) for each value of \( y \), which is a perfect square in the range we found in step 1.

So, in summary:

1. The range of \( Y \) is all non-negative integers from \( 0 \) to \( M^2 + 2M + 1 \).

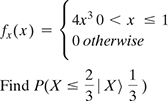
2. The pmf of \( Y \) is \( P\_Y(y) = p\_X(k-1) + p\_X(-k-1) \), where \( y = k^2 \) and \( k \) ranges from \( 0 \) to \( M+1 \).

2.Assuming X is a continuous random variable with PDF

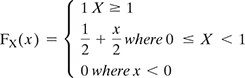


* + 1. Find EX and Var(X).
    2. Find *P*(*X* ≥ img).

1. If *X* is a continuous random variable with pdf

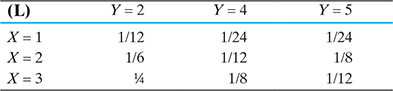


1. If *X*~Uniformimg and *Y* = sin(*X*), then find *fY*(*y*).
2. If X is a random variable with CDF



* + 1. What kind of random variable is *X*: discrete, continuous, or mixed?
    2. Find the PDF of *X*, f*X*(*x*).
    3. Find E(eX).
    4. Find P(*X* = 0|X≤0.5).

1. There are two random variables *X* and *Y* with joint PMF given in Table below
   * 1. Find *P*(*X*≤2, *Y*≤4).
     2. Find the marginal PMFs of *X* and *Y*.
     3. Find *P*(*Y* = 2|*X* = 1).
     4. Are *X* and *Y* independent?



6.A box containing 40 white shirts and 60 black shirts. If we choose 10 shirts (without replacement) at random, find the joint PMF of X and Y, where X is the number of white shirts and Y is the number of black shirts.

7.If A and B are two jointly continuous random variables with joint PDF

images

a. Find fX(a) and fY(b).

b. Are A and B independent of each other?

c. Find the conditional PDF of A given B = b, fA|B(a|b).

d. Find E[A|B = b], for 0 ≤ y ≤ 1.

e. Find Var(A|B = b), for 0 ≤ y ≤ 1.

8.There are 100 men on a ship. If Xi is the ith man's weight on the ship and Xi's are independent and identically distributed and EXi = μ = 170 and σXi = σ = 30. Find the probability that the men's total weight on the ship exceeds 18,000.

9.Let X1, X2, ……, X25 are independent and identically distributed. And have the following PMF

If Y = X1 + X2 + … + Xn, estimate P(4 ≤ Y ≤ 6) using central limit theorem.